Fall 2012 Math 211 Exam 1 Review Sheet

The exam will be in class on Monday, October 1, and will cover Sections 12.1-5 and 13.1-4. You will not be allowed use of a calculator or any other device other than your pencil or pen and some scratch paper. Notes are also not allowed. In kindness to your fellow test-takers, please turn off all cell phones and anything else that might beep or be a distraction.

Important topics:
- Computing dot and cross products; dot product of orthogonal vectors equals zero; magnitude squared of a vector equals the dot product with itself
- Triple scalar product
- Angle between vectors (memorize the formula!)
- Projection of a vector onto another vector (memorize the formula!)
- Finding unit vectors, vectors with prescribed length and angle, vectors perpendicular to given plane or pair of vectors.
- Parametric equations of a line
- Equation of a plane (given three points, given a point and a normal vector, etc)
- Unit tangent vector to a curve
- Vector function describing curve of intersection between two surfaces
- Intersection and angle between two curves given by vector-valued functions
- Angle between curves is the angle between their tangent vectors
- Angle between planes is the angle between their normal vectors
- Parametric equations for the tangent line to a curve
- Curvature and arc length
- Unit tangent, unit normal, and unit binormal vectors
- Position, velocity, and acceleration (including normal and tangential components)

Review exercises from Chapter 12 (p. 859): 1,5,6,11a,13,15-21,24,25
Review exercises from Chapter 13 (p. 898): 3,5,9,11,13,15,17,21

If you feel confident in working the review problems and the practice exam (on the back) under the exam rules, then you should be well prepared for the exam. If you have problems with certain topics or want more practice, feel free to stop by my office or send an email. You can also go to the Q-Center for drop-in tutoring (1-5 and 7-9pm).

Answers to even problems from Chapter 12 review:

6. $\pm \langle 7,2,-1 \rangle / (3\sqrt{6})$

16. $x=1+3t,\ y=2t,\ z=-1+t.$

18. $x+4y-3z=6.$

20. $6x+9y-z=26.$

24. Neither parallel nor perpendicular. Angle satisfies $\cos\theta = -5/\sqrt{87}$.

Practice Exam 1

1. Find an equation describing the plane that passes through the point (1,2,-2) that contains the line $x=2t$, $y=3-t$, $z=1+3t$.

2. Find the point of intersection and the angle between the two lines given by the vector-valued functions $\mathbf{r}(t) =\ <1-2t,3t,t>$ and $\mathbf{u}(s) =\ <1+s,-2s,5s>$.

3. Find parametric equations for the tangent line to the curve of intersection of the surfaces given by $z = x^2 + y^2$ and $y = 3 - x$ at the point (2,1,5).

4. Prove that if a particle is traveling at a constant speed, then its velocity vector $\mathbf{r}'(t)$ is always perpendicular to its acceleration vector $\mathbf{r}''(t)$.

5. Find the unit tangent vector $\mathbf{T}(t)$ to the curve $\mathbf{r}(t) = \left\langle 2\cos(t),\ \sqrt{5}t,\ 2\sin(t) \right\rangle$ and the arc length of the curve for $0 \le t \le \pi$.

6. Find $\mathbf{T}(t)$, $\mathbf{N}(t)$, $\mathbf{B}(t)$, and curvature κ for the curve $\mathbf{r}(t) = \left\langle 3\cos(t),\ 3\sin(t),\ 4t \right\rangle$.

Answers:
1. $6x+9y-z=26$
2. $(1,0,0)$, $\cos^{-1}(\sqrt{3}/\sqrt{140})$ or $\cos^{-1}(-\sqrt{3}/\sqrt{140})$, depending on which angle you choose; the first choice is more natural since the angle is less than $90°$
3. $x=2+t$, $y=1-t$, $z=5+2t$
4. $\left\| \mathbf{r}'(t) \right\|^2 = \mathbf{r}'(t) \cdot \mathbf{r}'(t) = C$. Taking a derivative gives $2\mathbf{r}'(t) \cdot \mathbf{r}''(t) = 0$, so the first and second derivatives are orthogonal.
5. Arc length is $s=3\pi$.
6. Curvature is $\kappa=3/25$.

More problems for practice (no solutions available):

1. Consider the two lines given by $\mathbf{r}(t) = <t,\ 3-3t,\ -2-t>$ and
   $\mathbf{u}(s) = <1+s,\ 4+s,\ s-1>$.
   (a) Find the point of intersection of these two lines.
   (b) Find an equation for the plane determined by these two lines. Please simplify the
       equation to linear form $ax+by+cz=d$.

2. Prove that if $\|\mathbf{r}(t)\|$ is constant, then $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}'(t)$ for all t.

3. Consider the helix $\mathbf{r}(t) = <4\cos t, 4\sin t, 3t>$.
   (a) Reparametrize the helix in terms of arc length s, measured from the point $(4,0,0)$.
   (b) Use part (a) to find the point on the curve that is a distance $15\pi/2$ units along the
       curve from the point $(4,0,0)$, in the direction of increasing t.
   (c) Calculate the curvature of this curve using the definition of curvature (the formula
       involving a derivative with respect to arc length s).

4. Find all unit vectors parallel to the line of intersection of the planes $x+2y+z-1=0$ and
   $x-y+2z+7=0$.

5. Find an equation for the plane that contains both of the lines
   $\mathbf{r}_1(t) = <t-1,\ t+2,\ 1-t>$ and $\mathbf{r}_2(s) = <1-4s,\ 1+2s,\ 2-2s>$.

6. Find a parameterization $\mathbf{r}(t)$ of the curve of intersection of the cylinder $x^2+y^2=4$ and
   the plane $z=1-x$. Prove that the acceleration vector $\mathbf{r}''(t)$ always lies parallel to the
   plane $z=1-x$.

7. Is the line described by $x=1-2t$, $y=2+5t$, $z=-3t$ parallel to the plane $2x+y-z=8$? If so,
   does the line lie in the plane? Explain your reasoning.

8. Is the line described by $x=1-2t$, $y=2+5t$, $z=-3t$ parallel to the plane $x+y+z=4$? If so, does
   the line lie in the plane? Explain your reasoning.

9. Find the equation of the tangent line to the curve $\mathbf{r}(t) = <t^3,\ t^2-1,\ 2t>$ at the point
   $(1,0,2)$.