Math 211 Final Exam Spring 2014

Read This First!

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.

- You need NOT simplify algebraically complicated answers. However, numerical answers such as $\sin \frac{\pi}{6}$, $\arctan (\sqrt{3})$, $4^{\ln 4}$, $\ln e^7$, $e^{-\ln 5}$, $e^{3 \ln 3}$, or $\cosh (\ln 3)$ should be simplified.

- Please read each question carefully. *Show all of your work and justify* all of your answers. (You may use the backs of pages for additional work space.)

Grading - For Administrative Use Only

<table>
<thead>
<tr>
<th>Question:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points:</td>
<td>6</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>6</td>
<td>8</td>
<td>100</td>
</tr>
<tr>
<td>Score:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Find the angle between \( u = \langle 2, 3, 1 \rangle \) and \( v = \langle 4, 1, 2 \rangle \).

2. Convert the following integral from rectangular to cylindrical coordinates. **DO NOT INTEGRATE.**

\[
\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-x^2-y^2}^{x^2+y^2} 21xy^2 \, dz \, dy \, dx
\]

3. Find the volume of the parallelepiped determined by \( u = \langle 2, 2, -4 \rangle \), \( v = \langle -2, 0, -2 \rangle \), and \( w = \langle 4, 3, -4 \rangle \).

4. Find the equation of the plane tangent to the surface

\[
z = \ln (2x + y)
\]

at the point \((-1, 3)\).

5. Find the volume of the region cut from the solid sphere \( \rho \leq 1 \) by the half planes \( \theta = 0 \) and \( \theta = \frac{\pi}{6} \) in the **first octant**.

6. A function is called ‘Harmonic’ if \( f_{xx} + f_{yy} + f_{zz} = 0 \). Show that the function

\[
f (x, y, z) = 7e^x + 2y \sin \left( z\sqrt{5} \right)
\]

is Harmonic.

7. Find the centroid of the triangular region cut from the **second quadrant** by the line \( y - x = 4 \).

8. For each of the following, find the limit or show that the limit does not exist.

(a) \( \lim_{(x,y) \to (4,0)} \frac{xy - 4y}{(x - 4)^2 + y^2} \)

(b) \( \lim_{(x,y) \to (0,0)} \frac{x^2 - 3y^3}{\sqrt{x^2 + y^2}} \)

9. Find the work done by a force field \( F = xyi + yzj + xzk \) from \((0, 0, 0)\) to \((1, 1, 1)\) over the path given by \( r(t) = ti + t^2j + t^4k \).
10. Consider the vector field \( F = (2xy^4 - \cos y) \mathbf{i} + (4x^2y^3 + 1 + x \sin y) \mathbf{j}. \)
   (a) Show that the vector field is conservative. \([4]\)
   (b) Find a potential function corresponding to \( F. \) \([4]\)
   (c) Evaluate the integral
   \[
   \int_C (2xy^4 - \cos y) \, dx + (4x^2y^3 + 1 + x \sin y) \, dy
   \]
   where \( C \) is a smooth curve from \((3, 1)\) to \( \left(2, \frac{\pi}{2}\right)\). \([4]\)

11. Given \( f(x, y) = \sqrt{29 - x^2 - y^2}, \) sketch the level curves that pass through the points \((2, -3, 4)\) and \((1, 1, 3\sqrt{3})\). Make sure to label your axes and tick marks. \([6]\)

12. Consider the function \( f(x, y) = x^2 + 4y^2. \)
   (a) Find the directional derivative of \( f \) at the point \((3, 1)\) in the direction of the vector \( \langle 1, -1 \rangle. \) \([6]\)
   (b) In what direction is the directional derivative greatest at \((3, 1)\)? \([2]\)