Amherst College
Department of Mathematics

Math 211 | Final Exam | Spring 2014

Name:____

Read This First!

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or other aids are permitted.
- You need *NOT* simplify algebraically complicated answers. However, numerical answers such as $\sin\frac{\pi}{6}$, $\arctan\left(\sqrt{3}\right)$, $4^{3/2}$, $e^{\ln 4}$, $\ln e^7$, $e^{-\ln 5}$, $e^{3\ln 3}$, or $\cosh(\ln 3)$ should be simplified.
- Please read each question carefully. *Show all of your work* and *justify* all of your answers. (You may use the backs of pages for additional work space.)

Grading - For Administrative Use Only

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	6	8	8	8	10	8	8	10	8	12	6	8	100
Score:													

1. Find the angle between $u = \langle 2,3,1\rangle$ and $v = \langle 4,1,2\rangle$. [6]

2. Convert the following integral from rectangular to cylindrical coordinates. [8]
DO NOT INTEGRATE.

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-x^2-y^2}^{x^2+y^2} 21xy^2 dz\, dy\, dx$$

3. Find the volume of the parallelepiped determined by $u = \langle 2,2,-4\rangle$, $v = \langle -2,0,-2\rangle$, and $w = \langle 4,3,-4\rangle$. [8]

4. Find the equation of the plane tangent to the surface [8]

$$z = \ln(2x+y)$$

at the point $(-1,3)$.

5. Find the volume of the region cut from the solid sphere $\rho \leq 1$ by the half planes $\theta = 0$ and $\theta = \dfrac{\pi}{6}$ in the **first octant**. [10]

6. A function is called 'Harmonic' if $f_{xx} + f_{yy} + f_{zz} = 0$. Show that the function [8]

$$f(x,y,z) = 7e^{x+2y}\sin\left(z\sqrt{5}\right)$$

is Harmonic.

7. Find the centroid of the triangular region cut from the **second quadrant** by the line $y - x = 4$. [8]

8. For each of the following, find the limit or show that the limit does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(4,0)} \frac{xy-4y}{(x-4)^2+y^2}$ [5]

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-3y^3}{\sqrt{x^2+y^2}}$ [5]

9. Find the work done by a force field $F = xyi + yzj + xzk$ from $(0,0,0)$ to $(1,1,1)$ over the path given by $r(t) = ti + t^2 j + t^4 k$. [8]

10. Consider the vector field $F = \left(2xy^4 - \cos y\right) i + \left(4x^2y^3 + 1 + x\sin y\right) j$.

   (a) Show that the vector field is conservative. [4]

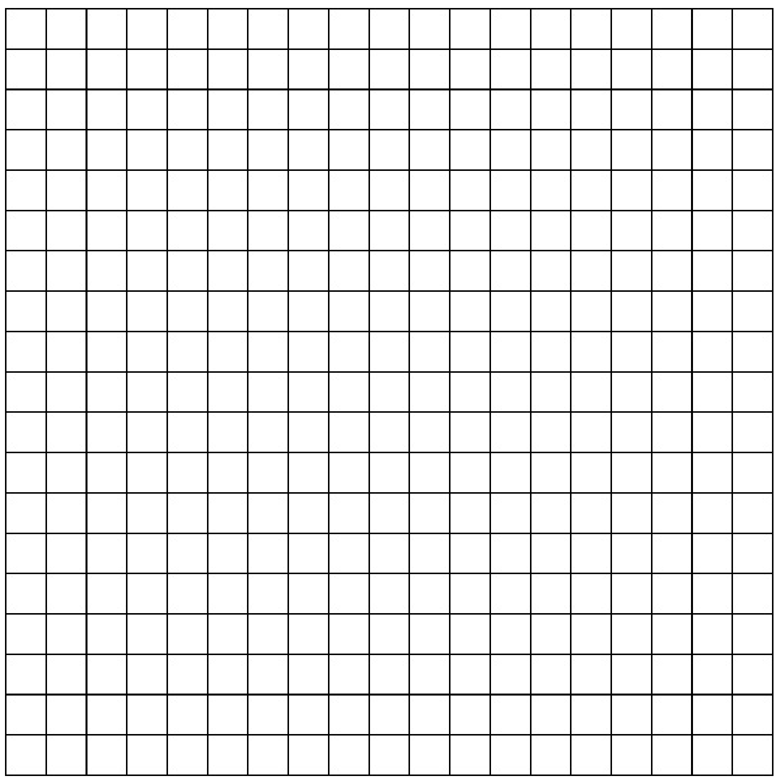
   (b) Find a potential function corresponding to F. [4]

   (c) Evaluate the integral [4]

$$\int_C \left(2xy^4 - \cos y\right) dx + \left(4x^2y^3 + 1 + x\sin y\right) dy$$

   where C is a smooth curve from $(3,1)$ to $\left(2, \dfrac{\pi}{2}\right)$.

11. Given $f(x,y) = \sqrt{29 - x^2 - y^2}$, sketch the level curves that pass through the points $(2,-3,4)$ and $\left(1,1,3\sqrt{3}\right)$. Make sure to label your axes and tick marks. [6]



12. Consider the function $f(x,y) = x^2 + 4y^2$.

   (a) Find the directional derivative of f at the point $(3,1)$ in the direction of the vector $\langle 1, -1\rangle$. [6]

   (b) In what direction is the directional derivative greatest at $(3,1)$? [2]