Math 211, Section 01, Spring 2012

Final Exam, Friday, May 11, 2012

Instructions: Do all twelve numbered problems. If you wish, you may also attempt the three optional bonus questions. **Show all work**, including scratch work. Little or no credit may be awarded, **even when your answer is correct**, if you fail to follow instructions for a problem or fail to **justify your answer.** Simplify your answers whenever possible, and **write legibly**. If you have time, check your answers. No calculators are allowed, but you may use one 8.5x11" sheet of notes.

1. (**10 points**). Find an equation for the line of intersection of the planes $2x - y + 5z = 1$ and $x - z = 3$.

2. **(15 points)** Let C be the curve in $\mathbb{R}^3$ parametrized by $\vec{r}(t) = \langle \cos t, \sin t, t^2 \rangle$, for $0 \le t \le \pi$.

2.a. Write down, **but do not evaluate**, a definite integral giving the arclength of C.

2.b. Compute $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x, y, z) = \langle yz, -xz, 6z \rangle$.

3. **(15 points)** Find the maximum and minimum values of the function

$$f(x, y) = 4x^2 y$$

on the ellipse $4x^2 + y^2 = 36$.

4. **(20 points)** Find and classify (as local minimum, local maximum, or saddle point) every critical point of the function $f(x, y) = x^2 y - 2x^2 - 6y^2 - 12y$.

5. **(20 points)** Let E be the solid bounded by the paraboloid $z = 2x^2 + 2y^2$ and the plane $z = 2$. Compute $\iiint_E z\, dV$.

6. **(20 points)** Find the volume of the solid bounded by the surfaces $y = 1 - x^2$, $y = 0$, $z = x$, and $z = 2$.

7. (**15 points**). Let $f(x, y) = \begin{cases} \dfrac{5y^3 - 2xy}{3x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

7.a. Compute $f_x(0, 0)$ and $f_y(0, 0)$.

7.b. Prove that f is not continuous at $(0, 0)$.

8. **(15 points)** Let C be the boundary of the triangle in the plane with vertices $(0, 0)$, $(1, 0)$, and $(0, 3)$, oriented **counterclockwise**, and let $\vec{F}(x, y) = \langle y^2 + \cos(x^2 - 3), 4xy \rangle$.
Compute $\int_C \vec{F} \cdot d\vec{r}$.

(over)

9. **(15 points)** Let $\vec{F}(x,y) = \langle x - \cos(2y), y^3 + 2x\sin(2y)\rangle$.

9.a. Show that $\vec{F}$ is conservative by finding a potential function for $\vec{F}$.

9.b. Let C be the curve parametrized by $\vec{r}(t) = \langle \sqrt{t^2+9}, e^{t^2-4t}\rangle$ for $0 \le t \le 4$.

Compute $\displaystyle\int_C \vec{F}\cdot d\vec{r}$.

(*Suggestion: Use part* (*a*) *somehow.*)

10. **(10 points)** Let $f(x,y)$ be a differentiable function, and suppose that:

$f_x(-1,1) = -2$	$f_x(-1,2) = 7$	$f_x(1,1) = 3$	$f_x(1,2) = 4$
$f_y(-1,1) = 2$	$f_y(-1,2) = -1$	$f_y(1,1) = 5$	$f_y(1,2) = -3.$

Let $h(s,t) = f(st - 2t, 3s - t)$. Compute $h_s(1,1)$.

11. **(20 points)** Let S be the closed surface consisting of the upper half of the sphere $x^2 + y^2 + z^2 = 4$ with $z \ge 0$, together with the disk $x^2 + y^2 \le 4$ in the xy-plane, oriented outward. Let $\vec{G}(x,y,z) = \langle xz, 3yz, x^2y\rangle$. Use the Divergence Theorem to compute the flux $\displaystyle\iint_S \vec{G}\cdot d\vec{S}$ of $\vec{G}$ through S.

12. **(25 points)** Let S be the portion of the surface $z = 4 - x^2 - y^2$ in the first octant, and let C be the boundary of S, oriented **clockwise** when viewed from above. (Note that C consists of three arcs, one in each of the three coordinate planes.) Use Stokes' Theorem to compute $\displaystyle\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y,z) = \langle x^5, xy, \sin z\rangle$.

OPTIONAL BONUS A. (2 points) Let C be the portion of the graph of $y = \sin x$ from the point $(0,0)$ to the point $(\pi, 0)$. Compute $\displaystyle\int_C (9x^2y^2 + y)\,dx + (6x^3y - \sin y)\,dy$.

OPTIONAL BONUS B. (2 points) Let (x_0, y_0) and (x_1, y_1) be two points in the plane for which $x_0, x_1 > 0$ and $x_0^2 + y_0^2 = x_1^2 + y_1^2 = 1$. Let C_0 be the straight line segment from (x_0, y_0) to $(0,-1)$, and let C_1 be the straight line segment from (x_1, y_1) to $(0,-1)$. Prove that $\displaystyle\int_{C_0} \frac{ds}{\sqrt{y_0 - y}} = \int_{C_1} \frac{ds}{\sqrt{y_1 - y}}$.

OPTIONAL BONUS C. (1 point) There are five nations that are **permanent** members of the United Nations Security Council. Name them.