Math 211 Spring 2017 Solutions

1. Curve of intersection: let $x=t$, so $y=\frac{1}{2}x^2=\frac{1}{2}t^2$, $z=3xy=\frac{3}{2}t^3$

$\boxed{\vec{r}(t)=\langle t, \frac{1}{2}t^2, \frac{3}{2}t^3\rangle}$

Tangent line at $(-2,2,-12)$ has $\langle a,b,c\rangle=\vec{r}\,'(-2)=\langle 1,-2,18\rangle$

Parametric eq's are $\boxed{x=-2+t,\ y=2-2t,\ z=-12+18t}$

2. (a) Pt of intersection: solve
$$5t+2=4-3s$$
$$-4t=12-s$$
$$t-7=-2s-1$$
(remember to change parameter in 2nd line to different letter)

Solution is $t=-2$, $s=4$, so point is $\boxed{(-8,8,-9)}$.

(b) Angle of intersection: $\theta=\cos^{-1}\dfrac{\langle 5,-4,1\rangle\cdot\langle -3,-1,-2\rangle}{\sqrt{42}\ \sqrt{14}}=\boxed{\cos^{-1}\left(\frac{-13}{14\sqrt{3}}\right)}$

(c) Plane containing both lines:

$\vec{n}$ is orthogonal to both lines, so can let

$\vec{n}=\langle 5,-4,1\rangle\times\langle -3,-1,-2\rangle=\langle 9,7,-17\rangle$

Plane is given by $9(x+8)+7(y-8)-17(z+9)=0$

$\boxed{9x+7y-17z=137}$

3. (a) $f_x(0,0)=\lim\limits_{h\to0}\dfrac{f(h,0)-f(0,0)}{h}=\lim\limits_{h\to0}\dfrac{\frac{3h^2-4h^3}{h^2}-3}{h}=\boxed{-4}$

$f_y(0,0)=\lim\limits_{h\to0}\dfrac{f(0,h)-f(0,0)}{h}=\lim\limits_{h\to0}\dfrac{\frac{3h^2}{h^2}-3}{h}=\boxed{0}$

(b) $\lim\limits_{(x,y)\to(0,0)}f(x,y)=\lim\limits_{r\to0}\left(3-\dfrac{4r^3\cos^3\theta}{r^2}\right)=3-\lim\limits_{r\to0}4r\cos^3\theta=3-0=3=f(0,0)$

Yes, f is continuous at $(0,0)$.

4. $f(x,y)=x\ln y+x$, at point $(1,1)$

(a) Increase most rapidly in direction of $\nabla f=\langle \ln y+1, \frac{x}{y}\rangle$

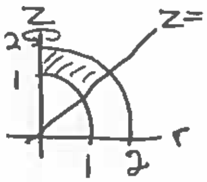
unit vector!

$\nabla f(1,1)=\langle 1,1\rangle\Rightarrow\boxed{\vec{u}=\langle 1/\sqrt{2}, 1/\sqrt{2}\rangle}$

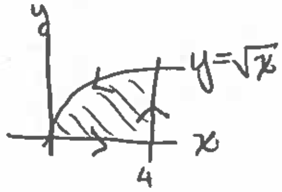
(b) Decrease most rapidly in direction of $-\nabla f$: $\boxed{\vec{u}=\langle -1/\sqrt{2}, -1/\sqrt{2}\rangle}$

(c) Directional derivative equals 0 if $\perp\nabla f$: $\boxed{\vec{u}=\langle 1/\sqrt{2}, -1/\sqrt{2}\rangle \text{ and } \langle -1/\sqrt{2}, 1/\sqrt{2}\rangle}$

(note $\langle a,b\rangle\perp\langle -b,a\rangle$)

5. 

$$\iiint_E z\,dV = \int_0^{2\pi}\int_0^{\pi/4}\int_1^2 \rho\cos\phi\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \boxed{\frac{15\pi}{8}}$$

6. 

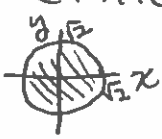
$$\int_C y^2dx + 6xy\,dy = \int_0^4\int_0^{\sqrt{x}} (6y-2y)\,dy\,dx \quad \text{(Green's Thm)}$$

$$= \boxed{16}$$

7. $\vec{F} = \nabla f$ where $f(x,y,z) = e^{xyz}$

$$\int_C \vec{F}\cdot d\vec{r} = f(\vec{r}(1)) - f(\vec{r}(0)) = f(1,-1,1) - f(0,0,0) = \boxed{\frac{1}{e} - 1}$$

8. Critical points: $f_x = y = 0$ and $f_y = x = 0 \Rightarrow (0,0)$ is inside region



For boundary, use Lagrange multipliers: $\nabla f = \lambda \nabla g$ + constraint

Solve $y = 2\lambda x \Rightarrow x = 4\lambda^2 x \Rightarrow \cancel{x=0}$ or $\lambda = \pm\frac{1}{2}$

$x = 2\lambda y$ $\quad\cancel{y=0}$ $\quad\Downarrow$

$x^2+y^2 = 2$ $\quad$ not on circle $\quad y = \pm x$

$\Downarrow$

$2x^2 = 2$

$\Downarrow$

$x = \pm 1$

List of points to check:

$f(0,0) = -1$

$f(1,1) = 0$

$f(1,-1) = -2$

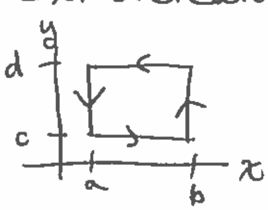
$f(-1,1) - 2$

$f(-1,-1) = 0$

Absolute max of 0 at $(1,1)$ & $(-1,-1)$

Absolute min of -2 at $(1,-1)$ & $(-1,1)$

Extra credit: $\int_C P\,dx = \int_a^b P(x,c)\,dx - \int_a^b P(x,d)\,dx$



$$-\iint_R \frac{\partial P}{\partial y}\,dA = -\int_a^b\int_c^d \frac{\partial P}{\partial y}\,dy\,dx$$

$$= -\int_a^b (P(x,d) - P(x,c))\,dx \quad \text{by Fund. Thm of Calculus}$$

$$= \int_a^b P(x,c)\,dx - \int_a^b P(x,d)\,dx$$

$$= \int_C P\,dx$$