Spring 2019 Math 211 Exam 1 Review Sheet

The exam will be in class on Monday, February 25, and will cover Sections 12.1-5 and 13.1-3. You will not be allowed use of a calculator or any other device other than your pencil or pen and some scratch paper. Notes are also not allowed. In kindness to your fellow test-takers, please turn off all cell phones and anything else that might beep or be a distraction.

Important topics:

- Computing dot and cross products; dot product of orthogonal vectors equals zero; magnitude squared of a vector equals the dot product with itself
- Angle between vectors (memorize the formula!)
- Projection of a vector onto another vector (memorize the formula!)
- Finding unit vectors, vectors with prescribed length and angle, vectors perpendicular to given plane or pair of vectors.
- Parametric equations of a line
- Equation of a plane (given three points, given a point and a normal vector, etc)
- Unit tangent vector and unit normal vector to a curve at a point
- Vector function describing curve of intersection between two surfaces
- Intersection and angle between two curves given by vector-valued functions
- Angle between curves is the angle between their tangent vectors
- Angle between planes is the angle between their normal vectors
- Parametric equations for the tangent line to a curve
- Curvature and arc length

Review exercises from Chapter 12 (p. 859): 1,5,6,11a,15,17-21,24,25
Review exercises from Chapter 13 (p. 898): 3,5,9,11,12,17

If you feel confident in working the review problems and the practice exam (on the back) under the exam rules, then you should be well prepared for the exam. If you have problems with certain topics or want more practice, feel free to stop by my office or send an email. You can also go to the Q-Center for drop-in tutoring.

Answers to even problems from Chapter 12 review:
6. $\pm\langle 7,2,-1\rangle/(3\sqrt{6})$
16. $x=1+3t$, $y=2t$, $z=-1+t$.
18. $x+4y-3z=6$.
20. $6x+9y-z=26$.
24. Neither parallel nor perpendicular. Angle satisfies $\cos\theta=-5/\sqrt{87}$.

Answer to #12 from Chapter 13 review: 3/16 and 4/9

Practice Exam

1. Find an equation describing the plane that passes through the point (1, 2, -2) that contains the line x=2t, y=3-t, z=1+3t.

2. Find the point of intersection and the angle between the two lines given by the vector-valued functions $\mathbf{r}(t) = < 1-2t, 3t, t >$ and $\mathbf{u}(s) = < 1+s, -2s, 5s >$.

3. Find parametric equations for the tangent line to the curve of intersection of the surfaces given by $z = x^2 + y^2$ and $y = 3 - x$ at the point (2, 1, 5).

4. Prove that if a particle is traveling at a constant speed, then its velocity vector $\mathbf{r}'(t)$ is always perpendicular to its acceleration vector $\mathbf{r}''(t)$.

5. Find the unit tangent vector $\mathbf{T}(t)$ to the curve $\mathbf{r}(t) = \left\langle 2\cos(t),\ \sqrt{5}t,\ 2\sin(t) \right\rangle$ and the arc length of the curve for $0 \le t \le \pi$.

Partial answers:

1. 6x+9y-z=26
2. (1,0,0), $\cos^{-1}(\sqrt{3}/\sqrt{140})$ or $\cos^{-1}(-\sqrt{3}/\sqrt{140})$, depending on which angle you choose; the first choice is more natural since the angle is less than 90°
3. x=2+t, y=1-t, z=5+2t
4. $\|\mathbf{r}'(t)\|^2 = \mathbf{r}'(t) \cdot \mathbf{r}'(t) = C$. Taking a derivative gives $2\mathbf{r}'(t) \cdot \mathbf{r}''(t) = 0$, so the first and second derivatives are orthogonal.
5. Arc length is s=3π.

Another Practice Exam

1. Consider the two lines given by

$$\mathbf{r}(t) = \langle 3t+1, t, 2t+1\rangle \text{ and } \mathbf{u}(t) = \langle t, 2t-2, 2-t\rangle.$$

   (a) Find the point of intersection of these two lines.
   (b) Find the angle between these two lines (leave answer in exact form).
   (c) Find an equation for the plane that contains both of these lines. Please simplify the equation to the form $ax+by+cz=d$ and divide out any common factors.

2. Find parametric equations for the line that is tangent to the curve

$$\mathbf{r}(t) = \langle 1-t^2, 5t, 2t^3\rangle$$

   at the point (0, 5, 2).

3. Find a parameterization $\mathbf{r}(t)$ of the curve of intersection of the cylinder $x^2+y^2=4$ and the paraboloid $z=9x^2+9y^2$. Determine the unit tangent vector $\mathbf{T}(t)$ and unit normal vector $\mathbf{N}$(t) for this curve.

4. Prove that if $\|\mathbf{r}(t)\|$ is constant, then $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}'(t)$ for all t.

5. Find all unit vectors parallel to the line of intersection of the planes $x+2y+z-1=0$ and $x-y+2z+7=0$.

Partial answers:

1. (1,0,1), $cos^{-1}\left(\frac{\sqrt{3}}{2\sqrt{7}}\right)$cos^{-1}(√3/2√7), x-y-z=0
2. x=-2t, y=5+5t, z=2+6t
3. $\mathbf{T}(t) = \langle -\sin(t), \cos(t), 0\rangle$, $\mathbf{N}(t) = \langle -\cos(t), -\sin(t), 0\rangle$
4. Take derivative of $\|\mathbf{r}(t)\|^2 = C^2$ (where C is some constant) to show that $\mathbf{r}(t)\cdot\mathbf{r}'(t) = 0$
5. $\pm\langle 5, -1, -3\rangle/\sqrt{35}$

A few more problems for practice (no solutions available):

1. Consider the two lines given by $\mathbf{r}(t) = < t,\ 3-3t,\ -2-t >$ and $\mathbf{u}(s) = < 1+s,\ 4+s,\ s-1 >$.
   (d) Find the point of intersection of these two lines.
   (e) Find an equation for the plane determined by these two lines. Please simplify the equation to linear form $ax+by+cz=d$.

2. Find $\mathbf{T}(t)$, $\mathbf{N}(t)$, and curvature κ(t) for the curve $\mathbf{r}(t) = \langle 3\cos(t),\ 3\sin(t),\ 4t \rangle$.

3. Find an equation for the plane that contains both of the lines $\mathbf{r}_1(t) = < t-1,\ t+2,\ 1-t >$ and $\mathbf{r}_2(s) = < 1-4s,\ 1+2\mathrm{s},\ 2 - 2\mathrm{s} >$.

4. Find a parameterization $\mathbf{r}(t)$ of the curve of intersection of the cylinder $x^2+y^2=4$ and the plane $z=1-x$. Prove that the acceleration vector $\mathbf{r}''(t)$ always lies parallel to the plane $z=1-x$.

5. Is the line described by $x=1-2t$, $y=2+5t$, $z=-3t$ parallel to the plane $2x+y-z=8$? If so, does the line lie in the plane? Explain your reasoning.

6. Is the line described by $x=1-2t$, $y=2+5t$, $z=-3t$ parallel to the plane $x+y+z=4$? If so, does the line lie in the plane? Explain your reasoning.

7. Find the equation of the tangent line to the curve $\mathbf{r}(t) = < t^3,\ t^2-1,\ 2t >$ at the point (1,0,2).

8. Determine $\mathbf{T}(t)$ and $\mathbf{N}(t)$ for the curve $\mathbf{r}(t) = \langle t - \sin t, 1 - \cos t \rangle$ at $t = \pi$. Draw a rough sketch of the curve and draw the tangent and normal vectors at that point, indicating their relationship to each other and to the curve.

9. Consider the plane described by the equation $4x - 5y + z = -2$, and another plane which passes through the points $(1, 1, 1)$, $(0, 1, -1)$, and $(4, 2, -1)$.
   a. Find a vector normal to the first plane.
   b. Find an equation describing the second plane.
   c. Find the angle between the planes.
   d. Find a point that lies on both planes.
   e. Find parametric equations for the line of intersection of the two planes.