Spring 2019 Math 211 Exam 3 Review Sheet

The exam will be in class on Monday, April 29, and will cover Sections 15.2-10 and 16.2. You will not be allowed use of a calculator or any other device other than your pencil or pen and some scratch paper. Notes are also not allowed. In kindness to your fellow test-takers, please turn off all cell phones and anything else that might beep or be a distraction.

Important topics:

- Iterated integrals (double and triple integrals)
- Finding volume of a solid region
- Finding mass and center of mass given a density function
- Finding average value of a function in a region (2D or 3D)
- Integrals in polar, cylindrical and spherical coordinates
- Surface area
- Basic integration rules like u-substitution (but you do not need to know integration by parts for the exam)
- Computation of line integrals

Review exercises from Chapter 15: 5,9,13,17,21,25,27,31,33,39,41,42,47 (#42: $\frac{64\pi}{9}$)

If you feel confident in working the review problems under the exam rules, then you should be well prepared for the exam. If you have problems with certain topics or want more practice, feel free to stop by my office or send an email.

Practice Exam Problems

1. Evaluate the line integral $\int_C (x-y)dx + (y-z)dy + z\,dz$ for the line segment from (0,0,0) to (1,4,4).

2. Evaluate the line integral of the function $f(x,y,z) = x^2+y^2+z^2$ over the path given by $\boldsymbol{r}(t) = \langle \cos(t), \sin(t), t\rangle$ for $0 \le t \le \pi$.

3. Sketch the region of integration, then reverse the order of integration:

   a. $\displaystyle\int_0^1 \int_{1-x}^{1-x^2} f(x,y)\,dy\,dx$

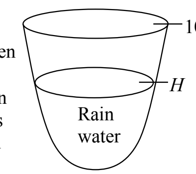
   b. $\displaystyle\int_0^{\ln 2} \int_{e^y}^{2} f(x,y)\,dx\,dy$

4. Find the volume of the part of the sphere $x^2+y^2+z^2=2$ that lies inside the paraboloid $z=x^2+y^2$.

5. Evaluate the triple integral $\iiint_E \sqrt{x^2+y^2+z^2}\,dV$, where E is the region between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ with $z \ge 0$ (above the *xy*-plane).

6. Evaluate $\iiint_E \left(x^2+y^2+z^2\right)^{5/2} dV$ where E is bounded by the *xz*-plane and the hemispheres $y=\sqrt{9-x^2-z^2}$ and $y=\sqrt{16-x^2-z^2}$.

7. Suppose you want to calibrate a bowl shaped like a paraboloid ($z=x^2+y^2$ for z=0 to 10 inches) to be a rain gauge. For example, if it "rained 2 inches," then the volume of rain collected in the bowl would be 2 inches times the area of the opening, 10π in^2, and you would mark the resulting height of the water in the bowl as the "2 inches of rain" level. Derive a general formula that relates the number of inches of rain R to the resulting height H of water collected in the bowl.



8. Find the center of mass of the solid below the paraboloid $z=2-x^2-y^2$ and above the cone $z=\sqrt{x^2+y^2}$ if the density is constant.

9. Find the volume of the part of the sphere $x^2+y^2+z^2=a^2$ that lies within the cylinder $x^2+y^2=ax$ and above the *xy*-plane, where a>0.

10. Evaluate the triple integral $\iiint_E z^2\,dV$, where E lies between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$.

11. Find the volume of the solid region bounded by y=0, y=3, $z=x^2$, and $z=2-x^2$.

12. Use polar or cylindrical coordinates to find the volume of the solid inside the sphere $x^2+y^2+z^2=9$ and above the cone $z=\sqrt{x^2+y^2}$.

13. Use spherical coordinates to find the volume of the solid inside the sphere $x^2+y^2+z^2=9$ and above the cone $z=\sqrt{x^2+y^2}$.

14. Find the average value of $f(x,y,z)=\sqrt{x^2+y^2+z^2}$ inside the sphere $x^2+y^2+z^2=1$.

15. A 1-inch radius cylindrical hole is drilled through the center of a ball with 2-inch radius. Find the volume of the core that was removed from the ball.

16. Find the surface area of the ellipsiodal-shaped region cut from the plane $z=cx$ (c is a constant) by the cylinder $x^2+y^2=1$.

17. Evaluate the integral $\int_{-2}^{0}\int_{-\sqrt{4-y^2}}^{0}\left(\frac{y}{(x^2+y^2)^{3/2}+1}\right)dxdy$.

Answers:

1. 13/2
2. $\sqrt{2}(\pi+\pi^3/3)$
3. (a) $\int_0^1\int_{1-y}^{\sqrt{1-y}} f(x,y)dxdy$ (b) $\int_1^2\int_0^{\ln x} f(x,y)dydx$
4. $\pi(8\sqrt{2}-7)/6$
5. $15\pi/2$
6. $\frac{\pi}{4}(4^8-3^8)$
7. $20R=H^2$
8. (0,0,11/10)
9. $(3\pi-4)a^3/9$
10. $124\pi/15$
11. $V=\int_0^3\int_{-1}^1(2-x^2-x^2)dxdy=8$
12. $V=\int_0^{2\pi}\int_0^{\frac{3}{\sqrt{2}}}\int_r^{\sqrt{9-r^2}} r\,dz\,dr\,d\theta=\int_0^{2\pi}\int_0^{\frac{3}{\sqrt{2}}}(\sqrt{9-r^2}-r)r\,dr\,d\theta=9\pi(2-\sqrt{2})$
13. $V=\int_0^{2\pi}\int_0^{\pi/4}\int_0^3\rho^2\sin\phi\,d\rho\,d\phi\,d\theta=9\pi(2-\sqrt{2})$
14. $Average=\frac{1}{V}\int_0^{2\pi}\int_0^{\pi}\int_0^1\rho^3\sin\phi d\rho d\phi d\theta=\frac{\pi}{\frac{4}{3}\pi}=\frac{3}{4}$
15. $V=\int_0^{2\pi}\int_0^1\left(\sqrt{4-r^2}-\left(-\sqrt{4-r^2}\right)\right)rdrd\theta=4\pi\left(\frac{8}{3}-\sqrt{3}\right)$
16. $\pi\sqrt{1+c^2}$
17. $-\frac{1}{3}\ln 9$

Please let me know if you find any errors in these answers. Thanks!

Practice Exam

1. (20pt) Sketch the region of integration and then evaluate the double integral

$$\int_0^1\int_{x^2}^1 x^3\sin(y^3)dydx$$

2. (20pt) Evaluate the line integral of the vector function $\mathbf{F}(x,y)=\langle x^2,xy\rangle$ along the quarter of the circle of $x^2+y^2=9$ going from (0,3) to (-3,0).

3. (20pt) Use **spherical coordinates** to find the volume of the solid above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=z$.

4. (20pt) Find the **surface area** of the part of the cone $z^2=a^2(x^2+y^2)$ between the planes z=1 and z=2, where a>0 is a constant.

5. (20pt) Use **polar coordinates** to find the average height of the hemisphere $z=\sqrt{a^2-x^2-y^2}$ above the disk $x^2+y^2\le a^2$, where a>0 is a constant. (Average height equals volume of the solid divided by area of its base. You can easily check your answer using standard geometry formulas for area and volume, but for full credit, evaluate the answer using a double integral involving polar coordinates.)

Partial answers:

1. (1-cos(1))/12
2. 0
3. $\frac{\pi}{8}$
4. $\frac{3\pi}{a}\sqrt{1+\frac{1}{a^2}}$
5. $\frac{2a}{3}$

Practice Exam (no solutions available)

1. Sketch the region of integration and then evaluate the double integral

$$\int_0^4 \int_{\sqrt{y}}^2 \sqrt{4x^2+5y}\, dx\, dy$$

2. Evaluate the line integral $\int_C xy\, ds$ where C is the quarter circle going counterclockwise from (0,3) to (-3,0).

3. Use **spherical coordinates** to find the volume of the solid inside the sphere $x^2+y^2+z^2=4$ and above the cone $z=\sqrt{x^2+y^2}$.

4. Set up an integral to find the volume of the solid in the 1st octant bounded by $x+y+z=4$ and $x+y+2z=4$. You do not need to evaluate this integral.

5. Find the center of mass of the solid bounded by the paraboloid $z=4-x^2-y^2$ and the cone $z=3\sqrt{x^2+y^2}$ with constant density k (you may use symmetry to deduce some of the coordinates of the center of mass).