

Math 211 div-grad-curl Practice

For all three problems, $f(x, y, z)$ is a real-valued function $\mathbb{R}^3 \rightarrow \mathbb{R}$, while $\mathbf{F}(x, y, z)$ is a vector-valued function $\mathbb{R}^3 \rightarrow \mathbb{R}^3$. Assume all partial derivatives exist and are continuous.

1. Prove that $\operatorname{div}(f\mathbf{F}) = f\operatorname{div}(\mathbf{F}) + \mathbf{F} \cdot \nabla f$.

2. Prove that $\operatorname{curl}(f\mathbf{F}) = f\operatorname{curl}(\mathbf{F}) + \nabla f \times \mathbf{F}$.