Math 211 Vector Functions Practice

1. Determine $\mathbf{r}(t)$ if $\mathbf{r}''(t) = \langle t^2 - 1, t + 1,\ t^3 \rangle$, $\mathbf{r}(0) = \langle 1,0,0 \rangle$, and $\mathbf{r}'(0) = \langle -1,1,0 \rangle$.

2. This exercise explores key relationships between a pair of lines. Consider a line described by the vector function $\boldsymbol{u}(s) = \langle 4 - 2s,\ s - 2,\ 1 + 3s \rangle$ and a second line that passes through $(-4, 2, 17)$ in the direction $\mathbf{v} = \langle -2, 1, 5 \rangle$.
   a. Find a direction vector $\langle a, b, c \rangle$ for the first line.
   b. Find a vector function $\mathbf{r}(t)$ describing the second line.
   c. Show that the two lines intersect at a single point by finding the values of s and t that result in the same point. State that point of intersection.
   d. Find the acute angle formed where the two lines intersect, that is, the acute angle between the lines' direction vectors.
   e. Find an equation for the plane that contains both of the lines described in this problem.

3. Let a and b be positive real numbers. The equation $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ describes an ellipse centered at (h, k) with a horizontal axis length of $2a$ and a vertical axis length of $2b$.
   a. Explain why the vector function $\boldsymbol{r}(t) = \langle a\cos(t), b\sin(t)\rangle$, $0 \leq t \leq 2\pi$, is a parameterization of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
   b. Find a parameterization of the ellipse $4x^2 + y^2 = 16$.
   c. Find a parameterization of the ellipse $\frac{(x+3)^2}{4} + \frac{(y-2)^2}{9} = 1$.
   d. Determine the *x*-*y* equation of the ellipse that is parameterized by

$$\boldsymbol{r}(t) = \langle 3 + 4\sin(2t), 1 + 3\cos(2t)\rangle.$$

4. If $\boldsymbol{r}(t) \neq \boldsymbol{0}$, show that $\frac{d}{dt}\|\boldsymbol{r}(t)\| = \frac{1}{\|r(t)\|}\boldsymbol{r}(t) \cdot \boldsymbol{r}'(t)$.