

Math 260 Exam 1 Review

Important concepts and methods:

- Geometric analysis of a 1st order autonomous DE
- Linear stability analysis
- Potential functions
- Saddle-node bifurcation
- Transcritical bifurcation
- Supercritical and subcritical pitchfork bifurcations

For the bifurcations, know the normal form and its bifurcation diagram for each of the 4 major types. You don't need to memorize the bifurcation theorems.

Given a DE, be able to determine the fixed points and their stability. If a parameter is involved, be able to sketch the bifurcation diagram.

Example exam problems for practice (you may well see them on the exam itself!):

1. State the normal form for a saddle-node bifurcation and sketch its bifurcation diagram.
2. State the normal form for a transcritical bifurcation and sketch its bifurcation diagram.
3. State the normal form for a supercritical pitchfork bifurcation and sketch its bifurcation diagram.
4. For the DE $\dot{x} = 1 - x^2$, compare the 3 types of stability analysis:
 - a. Apply *geometric analysis* to sketch the phase portrait, showing fixed points and flow between them.
 - b. Apply *linear stability analysis* and confirm that the results agree with (a).
 - c. Find and sketch the *potential function* $V(x)$. Identify the stability of the fixed points according to the potential function.

For the following DEs, find the fixed points, determine their stability and how it depends on the parameter value (sketch the different possible cases of phase portraits), and then draw the bifurcation diagram. Identify the type of bifurcation, if it is one of the standard types.

5. $\dot{x} = rx + x^2$
6. $\dot{x} = 1 + rx + x^2$
7. $\dot{x} = x - rx^2$
8. $\dot{x} = x(1 - x) - r$
9. $\dot{x} = rx - x^3$
10. $\dot{x} = r - 3x^2$