Differential Equations Final Project

Math 260 Prof. Leise

Goal: To delve deeply into a topic of interest in differential equations by finding and studying an article or part of a book on that topic and then writing a report, which should include some mathematical analysis or computations.

Suggested topics (you are not limited to these; an internet search will yield many topics):

- Full proof of the existence/uniqueness theorem, with some historical background into the mathematicians involved in developing the theory (e.g., Picard)
- Derivation of Runge-Kutta method or other numerical methods, with some historical background about who Runge and Kutta were
- Deeper look at normal forms for bifurcations
- Solution methods for specific types of DEs, for example, the Laplace transform method for linear DEs with forcing terms and the notion of transfer functions
- Application to lasers (e.g., articles by H. Haken)
- Synchronized applause
- Self-organization and emergent properties of networks, including applications to humans and social groups (e.g., J.A.S. Kelso) or other applications
- Epidemiological models (SIR and variants)
- Ecological models
- Strogatz's circadian rhythms/sleep-wake model (or Kronauer's model)
- John Tyson's model of the cell cycle (process of cell division) based on bifurcation theory
- DEs on fractals, see book by Strichartz in the AC Science Library
- Deeper look at catastrophe theory (for example, the different basic types and their connection to Lie groups)
- Other application of DEs, including economics, physics, biology, social sciences

Feel free to talk to me at any point about finding sources and what material should be included in the final report.

Timeline:

- Choose a topic by **4pm Monday April 15** and email me a proposal of what you want to do (a few sentences describing your proposed project).
- Submit outline of project and sources (by email is fine) by **4pm Wednesday, May 1**.
- Final report due **4pm Tuesday May 14**. Emailing me your file is fine.

Report guidelines: The report should be roughly 8 pages double-spaced, using Word, LaTeX, Mathematica (which has some nice document styles if you learn them), or some other appropriate format. The report should include significant mathematics (theoretical or computational), but may also include less technical explanations and relevant background. For example, if the topic is the system of Lorentz equations, the report could include the mathematics of how he derived the equations and why (mathematically) they display chaotic behavior, as well as some history of why Lorentz was studying those particular equations and why his results opened up the field of chaotic dynamical systems.

Sources: You should use at least three sources of information, which may include your textbook, other books, and scholarly articles. You may not use a website as a source of information (since websites often contain incorrect information), but searching the web may be helpful initially as an idea-generator of interesting topics and for basic information. Searching JSTOR and MathSciNet may also be helpful, in addition to a general 5 College library search (start looking for books and articles early in case you need to ask for an interlibrary loan or order an article to be delivered).

Your report should list **all sources** used in to writing your report. You may use any standard style to cite them, for example:

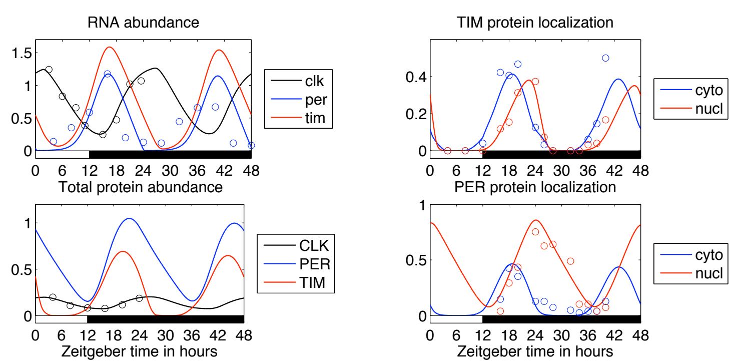
Baker, G.L., and Gollub, J.P. *Chaotic Dynamics: An Introduction*, Cambridge University Press, Cambridge, 1990.

Li, T.-y., and Yorke, J., “Period Three Implies Chaos.” *American Mathematical Monthly* **82** (1975), 985-992.

There are two purposes in citing your sources: first, to give credit to those who did the work and published it, and second, to enable readers to find these article or books if they want to read further about that topic.

When you refer to a source of information in the text of your report, cite that source using a standard style, as in the following examples:

<u>One author:</u> How fireflies oscillate in synchrony can be explained using a relatively simple nonlinear system (Strogatz, 1994).
<u>Two authors:</u> Tyson and Novak (2001) discovered a bifurcation that explains the cell cycle.
<u>More than two authors:</u> Tyson et al. (2004) found that something interesting occurred.



<u>If you copy a figure, cite the source in the caption:</u> Simulation of clock gene expression in Drosophila neurons (Leise and Moin, 2007).

Please recall that this project is officially 20% of your grade. This is your opportunity to learn about something of interest to you that involves differential equations. Feel free to propose a topic that is completely different from anything we have discussed, but also feel free to choose a project that delves deeply into a topic that we did discuss.