Parking Lot Problem   Name:

Please turn in a complete worksheet by Monday 4pm.

A university is studying their parking situation. Traffic has become congested in the three lots (A, B, and C) on campus, and an overflow lot (K) is going to be built half a mile from campus. The planners wish to understand the traffic flow among the lots in order to improve their plans.

Let's take a typical driver arriving on campus. Our job is to track how often he parks in each lot and how long he spends driving around the lots. The "states" will be cruising around lot A, B, or C, or parked in lot A, B, C, or K. We'll add a "P" to the letters A, B, and C to indicate that the driver is parked.

We will make the following assumptions, based on surveys of drivers parking on campus:

- After 3 minutes of looking for a parking spot in lot A, 1/5 of drivers continue driving around lot A, 4/15 park in lot A, 1/3 move on to lot B, and 1/5 move on to lot K.
- After 3 minutes of looking for a parking spot in lot B, 1/3 of drivers move on to lot A, 1/9 continue looking in lot B, 2/9 park in lot B, and 1/3 move on to lot C.
- After 3 minutes of looking for a parking spot in lot C, 1/3 move on to lot B, 1/5 of drivers continue driving around lot C, 4/15 park in lot C, and 1/5 move on to lot K.
- Once parked, the driver runs off to class and remains parked.
- Everyone who ends up in lot K, the overflow lot, parks there.

1. Find the transition matrix **T** given the probabilities of changing states after 3 minutes, with the starting state given by the column and the new state by the row. Observe that each column sum should equal 1.

	A	AP	B	BP	C	CP	KP
A	1/5	0					
AP	4/15	1					
B	1/3	0					
BP	0	0					
C	0	0					
CP	0	0					
KP	1/5	0					

Multiplying a vector of probabilities that a driver will be in each state by this transition matrix gives us the probabilities of being in each state 3 minutes later.

2. Currently the only two entrances to campus are by lots A and C. Assume drivers choose one randomly, so that half the time a driver tries lot A first and half the time (s)he tries lot C first. This means that our initial state vector will be $\mathbf{x}=[\frac{1}{2}\ 0\ 0\ 0\ \frac{1}{2}\ 0\ 0]^T$.

   (a) What is the probability that the driver is parked after 3 minutes? (Take the transition matrix **T** times the initial vector, and then add up the components of the resulting vector that correspond to being parked in any of the lots.)

   (b) What is the probability of being parked after two time steps?

   (c) What is the probability of being parked after 5 time steps (15 minutes)?

3. The command Eigensystem[A] gives a list of the eigenvalues of A followed by the respective eigenvectors.
   (a) What are the eigenvectors of the transition matrix **T** that correspond to eigenvalue=1? That is, what vectors satisfy **Tx**=**x**?

   (b) What is the meaning of these eigenvectors for our parking problem?

4. Look at large powers of **T**, like $\mathbf{T}^{20}$, times the starting state: **MatrixPower[T,20].x**. This gives the approximate long-term distribution of parking. What is the predicted distribution of parking among the lots A, B, C, and K? How big does the overflow lot K need to be, relative to the size of the regular lots, based on this model?

5. The equilibrium vector found in #4 (giving the eventual distribution of parking) should be an eigenvector of **T** with eigenvalue 1 (multiplying again by **T** doesn't change the vector, so **Tx**=**x**). According to your answer to #3, is this true? (Note that any linear combination of eigenvectors is also an eigenvector.)