Math 294 Exercises on Convex Functions

1. Give an example of a convex function $f : \mathbb{R} \to \mathbb{R}$ and sketch its epigraph.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a convex function. Prove that $g(x) = f(ax + b)$ is also a convex function for any real numbers a and b.

3. Exercise 8.1 on pages 287-8.

4. Prove that the negative entropy function $f(x) = x \log(x)$ is convex on the set of positive reals.

5. Prove that the log-sum-exp function $f(x_1, x_2) = \log(e^{x_1} + e^{x_2})$ is convex on $\mathbb{R}^2$. Sketch some level sets of this function, and sketch an example of a sub-level set S_α.

6. Prove that the function $f(x) = \|Ax - b\|_2$ is a convex function, where A is an m by n matrix, $b \in \mathbb{R}^m$, and $x \in \mathbb{R}^n$.