Math 294 Exercises on Subdifferentials and Coercive Functions

1. Find the subdifferential $\partial f(x)$ of the function $f(x) = |3x - 6|$ for $x \in \mathbb{R}$. Sketch the function and its subdifferential.

2. Find the subdifferential $\partial f(x)$ of the function $f(x) = \max\{|3x - 6|, |2x + 1|\}$ for $x \in \mathbb{R}$. Sketch the function and its subdifferential.

3. Find the subdifferential $\partial f(x)$ of the function $f(x) = |3x - 6| + |2x + 1|$ for $x \in \mathbb{R}$. Sketch the function and its subdifferential.

4. Find the subdifferential $\partial f(x)$ of the function $f(x) = \|Ax - b\|_2$ for $x \in \mathbb{R}^2$, where $b = [1\ 3]^T$ and
$$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}.$$

5. Is the function $f(x, y) = x^2 + 2xy + y^2$ coercive? Sketch a sublevel set S_α for some value of α and use that to confirm your answer.

6. Is the function $f(x, y) = x^4 + 2xy + y^4$ coercive? Sketch sublevel sets S_α for the values $\alpha = -\frac{1}{4}$, 0, and 1 (you can use graphing software). Find all points where the global minimum of f occurs.