Binomial distribution practice problems

1. A student is given a multiple choice exam with 10 questions, each question with five possible answers. Assume the student randomly chooses answers to the questions.

    a. What's the probability that they will get **exactly** 6 questions correct?

    b. What's the probability that they will get **at least** 6 questions correct?

2. The Everlasting Lightbulb company produces light bulbs which are packaged in boxes of 20 for shipment. Tests have shown that 4% of their light bulbs are defective.

    a. What is the probability that a box ready for shipment contains exactly 3 defective light bulbs?

    b. What is the probability that a box ready for shipment contains 3 or more defective light bulbs?

3. When sending messages over a network, there is a chance that the bits will be corrupted. A Hamming code allows for a 4 bit code to be encoded as 7 bits, with the advantage that if 0 or 1 bits are corrupted, then the message can be perfectly reconstructed. You are working on the Voyager space mission and the probability of any bit being lost in space is 0.1. How does reliability change when using a Hamming code?

    a. First suppose we don't use error correcting codes. Let $X \sim Binom(4,0.1)$. What is the probability of a corrupted message (at least one bit incorrect)?

    b. Now suppose we do use error correcting codes, so up to one bit incorrect will still be okay. Let $X \sim Binom(7,0.1)$. What is the probability of a corrupted message (at least two bits incorrect)?