Central Limit Theorem and Moment Generating Function Practice Problems

1. A senior researcher claims that adult male salamanders of a particular species have a mean length of 7 cm with a variance of 1 cm^2. A junior researcher has gathered the lengths of 64 adult male salamanders from the species and found a sample mean length of 7.3 cm. Does this result seem unusual given the senior researcher's claim? Explain.

2. Prove using mgfs that any linear function Y = aX + b, where $a \neq 0$, of a normally distributed random variable X is also normally distributed. Be sure to specify the parameters of the resulting normal distribution (read off Y's mgf).

3. Explain using the mgf for the geometric distribution why the mgf for the negative binomial random variable X ~ NegBinom(r, p) is

$$m_X(t) = \left(\frac{pe^t}{1-(1-p)e^t}\right)^r \quad \text{for } t < ln(1/(1-p))$$

4. Derive the mgf for X ~ Uniform(a,b). Obtain E[X] by taking the derivative of the mgf and then taking the limit as t goes to 0 using l'Hopital's rule.