Joint Distribution Practice Problems

Let $X$ and $Y$ be discrete random variables with joint probability mass function $P(X=x, Y=y)$. Equate the terms and expressions on the left below with their corresponding formulas on the right.

1. ______ $\text{Cov}(X,Y)$  
   A. $E(X^2) - [E(X)]^2$
2. ______ $E(Y)$  
   B. $P(Y = y \mid X = x) = P(X = x, Y = y) / P(X = x)$
3. ______ $\text{Var}(X)$  
   C. $P(X = x) = \sum_y P(X = x, Y = y)$
4. ______ $\text{Corr}(X,Y)$  
   D. $\sum_y yP(Y = y)$
5. ______ $E(X \mid Y=y)$  
   E. $\text{Var}(X) + \text{Var}(Y) - 2\text{Cov}(X,Y)$
6. ______ Conditional pmf of $Y \mid X$  
   F. $E[(X - \mu_x)(Y - \mu_y)]$
7. ______ $\text{Var}(X-Y)$  
   G. $\text{Cov}(X,Y)/[\text{SD}(X)\text{SD}(Y)]$
8. ______ Marginal of $X$  
   H. $\sum_x xP(X = x \mid Y = y)$
9. ______ $\text{Var}(Y)$  
   I. $E[(Y - \mu_y)^2]$

A. Suppose that $X$ and $Y$ have a discrete joint distribution for which the joint probability function is defined as follows: $f(x,y) = (x + y)/30$ for $x = 0,1,2$ and $y = 0,1,2,3$. Find the marginal probability mass functions for $X$ and $Y$, and determine if $X$ and $Y$ are independent.
B. A merchant stocks a certain perishable item. He knows that on any given day he will have a demand for two, three, or four of these items, with probabilities 0.2, 0.3, and 0.5 respectively. He buys the items at a cost of 1 dollar and sells them for 1.20 dollars. Any items left at the end of the day represent a total loss. How many items should he stock to maximize his expected profit?