

## Joint Distribution Practice Problems

Let  $X$  and  $Y$  be discrete random variables with joint probability mass function  $P(X=x, Y=y)$ . Equate the terms and expressions on the left below with their corresponding formulas on the right.

1. \_\_\_\_\_  $\text{Cov}(X, Y)$

A.  $E(X^2) - [E(X)]^2$

2. \_\_\_\_\_  $E(Y)$

B.  $P(Y = y | X = x) = P(X = x, Y = y) / P(X = x)$

3. \_\_\_\_\_  $V(X)$

C.  $P(X = x) = \sum_y P(X = x, Y = y)$

4. \_\_\_\_\_  $\text{Corr}(X, Y)$

D.  $\sum_y yP(Y = y)$

5. \_\_\_\_\_  $E(X | Y=y)$

E.  $V(X) + V(Y) - 2\text{Cov}(X, Y)$

6. \_\_\_\_\_ Conditional pmf of  $Y | X$

F.  $E[(X - \mu_X)(Y - \mu_Y)]$

7. \_\_\_\_\_  $V(X-Y)$

G.  $\text{Cov}(X, Y) / [SD(X)SD(Y)]$

8. \_\_\_\_\_ Marginal of  $X$

H.  $\sum_x xP(X = x | Y = y)$

9. \_\_\_\_\_  $V(Y)$

I.  $E[(Y - \mu_Y)^2]$

- A. Suppose that  $X$  and  $Y$  have a discrete joint distribution for which the joint probability function is defined as follows:  $f(x, y) = (x + y)/30$  for  $x = 0, 1, 2$  and  $y = 0, 1, 2, 3$ . Find the marginal probability mass functions for  $X$  and  $Y$ , and determine if  $X$  and  $Y$  are independent.

- B. A merchant stocks a certain perishable item. He knows that on any given day he will have a demand for two, three, or four of these items, with probabilities 0.2, 0.3, and 0.5 respectively. He buys the items at a cost of 1 dollar and sells them for 1.20 dollars. Any items left at the end of the day represent a total loss. How many items should he stock to maximize his expected profit?