Practice with conditional expectation and variance

1. Suppose X and Y have joint pdf $f(x,y)=x+y$ for $0<x<1$ and $0<y<1$, and 0 otherwise.
    a. Find the conditional distribution of Y given X.
    b. Find $E[Y \mid X=x]$ as a function of x.
    c. Explain how you could use the Law of Total Expectation to find E[Y] using part b.
    d. Write out the integral expression to compute $E[Y^3 - 5XY \mid X]$. Do not evaluate.

2. Suppose you recently acquired a new electronics system with 10 components. Assume all the components have equal probability of being defective in some way. You should assume the components may be treated independently (if a component you check has a defect, that doesn't affect the others in any way). However, the probability of being defective is known to vary based on the day of production. With no information about this probability, you decide to model the probability of being defective on a given day as Uniform(0,1).
   a. What is the expected value of the number of defective components in your system?
   b. What is the variance for the number of defective components in your system?

3. Suppose two random variables X and Y have a joint density given by

$$f(x, y) = e^{-y} \text{ for } 0 < x \leq y < \infty, \text{ and 0 otherwise.}$$

   a. Find the conditional distribution of X given Y.
   b. Find E[X | Y=y].
   c. Set up how you would find V[X | Y=y]. Do not evaluate.
   d. Let Z = E[X | Y]. What is the pdf for Z?