

## More practice with conditional expectation and variance

1. Suppose  $X$  and  $Y$  have the following joint pdf:  $f(x,y)=8xy$ ,  $0 < y < x < 1$ , and 0 otherwise.
  - a. Find the conditional distribution of  $Y$  given  $X$ .
  - b. Find  $E[Y | X=x]$  as a function of  $x$ .
  - c. Explain how you could use the Law of Total Expectation to find  $E[Y]$  using part b.
  - d. Write an integral for  $E[Y^2 - 3Y | X=x]$ . Do not evaluate.

2. Suppose  $X$  and  $Y$  are independent continuous random variables, not necessarily with the same distribution. Prove that  $E[Y] = E[Y \mid X]$ .
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
3. You have just purchased an “assembly required” piece of furniture. It contains 14 pieces which all came off an assembly line on the same day, and which all have equal probability of being defective in some way. You should assume the pieces may be treated independently (if a piece you check has a defect, that doesn't affect the probability of the others being defective). However, the probability of being defective varies from day to day, following a  $\text{Beta}(1,10)$  distribution. What is the expected value and standard deviation for the number of defective pieces in the set you just bought?

4. Suppose that in 2 neighboring counties, road repairs per week are classified as major if the amount of road that requires repairing is 3 miles or longer. Assume the repair lengths in both counties range between 0 and 4 miles (continuous, not discrete). Let  $X$  denote length of repairs in one county and  $Y$  denote length of repairs in the neighboring county for a given week. Assume the joint pdf of  $X$  and  $Y$  is given by  $f(x,y) = xy/64$  for  $0 \leq x,y \leq 4$ .
- What is the marginal pdf of  $X$ ?
  - Find the conditional pdf for  $Y$  given  $X$ .
  - What does your result in part b suggest about the relationship between  $X$  and  $Y$ ? How could you have deduced this relationship without computing the conditional pdf?
  - What is the probability road repairs for the second county are classified as major if  $X=1$  for the first county for a given week?
  - The two counties want to hire a single company for the repairs. The company will only handle combined jobs of up to 6 miles at a time for a given week before charging huge additional fees. Compute a meaningful probability and argue why you would or wouldn't recommend using this company for their repairs.
  - Simplify then set up appropriate integrals to evaluate  $E[X^2 + 2XY + Y^2 | X]$ .