

More conditional probability practice problems

1. An actuary is studying the prevalence of three health risk factors, denoted by A , B , and C , within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B , is one-third. What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A ?

5. For the year of 1973, the University of California at Berkeley was involved in legal action relating to the claim that there was gender discrimination in graduate school admissions (see paper by Bickel et al. 1975 for details). The table below contains school-wide admission/denied numbers for 1973 for the pool of 12763 applicants.

	Admitted	Denied	Total
Male	3738	4704	8442
Female	1494	2827	4321

- What is the probability a randomly selected applicant was female?
- What is the probability a randomly selected applicant was admitted?
- What is the probability a randomly selected applicant was admitted if you knew the applicant was female?
- Do you think this is strong evidence of gender discrimination in the graduate school admission process?

Here is the breakdown by some of the graduate school departments (only the largest 6 departments are shown out of 101 total departments). Fill in the missing conditional probabilities of the form $P(\text{admit male given dept. A}) = \%M.\text{admit}$ and $P(\text{admit female given dept. A}) = \%F.\text{admit}$.

Dept	M admit	M denied	M total	%M.admit	F admit	F denied	F total	%F.admit
A	512	313	825		89	19	108	
B	353	207	560	.63	17	8	25	.68
C	120	205	325	.37	202	391	593	.34
D	138	279	417		131	244	375	
E	53	138	191	.28	94	299	393	.24
F	22	351	373	.06	24	317	341	.07

What do you think now about the claims of gender discrimination? What do you think was going on here?

Situations where probabilities seemingly “reverse” when a third variable (department) are taken into account are examples of a phenomenon known as Simpson’s Paradox. In this case, it was determined that the “reversal” when department was taken into account was due to the fact that women had applied in larger numbers to departments that were harder to get into (fewer slots in proportion to number of applicants makes it harder even among qualified applicants) while men were applying to departments that were relatively easier to get into, resulting in a lower overall admission rate for women even if the department level bias appears to be in favor of women!

6. The following table contains data on number of eggs hatched and not for 3 different temperature settings for python eggs, as recorded by scientists studying environmental impact of temperature changes on python reproduction. As you answer the questions below, be sure to use clear notation for your events and probabilities.

	Hatched	Didn't hatch	Total
Cold temp	16	11	27
Neutral temp	38	18	56
Hot temp	75	29	104
Total	129	58	187

- a. What is the probability a randomly selected egg from this experiment hatched?
- b. Given each temperature setting (do a calculation for cold, neutral, and hot), what is the probability a randomly selected egg from that level hatched?
- c. Verify Bayes' Rule for $P(\text{Neutral} \mid \text{Hatch})$ holds. In other words, apply Bayes' Rule to find the probability and verify it matches what you obtain using the table directly.