More conditional probability practice problems

1. An actuary is studying the prevalence of three health risk factors, denoted by A, B, and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is one-third. What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A?

- 2. A data set collected on wolf skulls includes measurements of the crown length (CL) of the upper first molar of each skull. CL is categorized as short if below 17.25mm and as long otherwise. 35% of the wolf skulls in the collection are from females and only 1 in 3 female skulls has a long CL. For male skulls, 7 out of 8 have a long CL. One wolf skull is drawn randomly from the collection.
 - a. What is the probability of obtaining a skull with a long CL?

b. What is the probability it is a male skull given that it has a short CL?

c. What is the probability it is a female skull given that it has a long CL?

- 3. Two six-sided dice are rolled and the upper faces observed.
 - a. What is the probability of rolling doubles?
 - b. What is the probability the sum of the upper faces is 8?
 - c. What is the probability of rolling doubles if it is known that the sum is 8?
 - d. What is the probability the sum is 11 if it is known that doubles were rolled?
 - e. What is the probability that the sum is 11 or doubles were rolled?
 - f. What is the probability that doubles were rolled if it is known that the sum is even?

4. A study to see if a liver scan can detect abnormal liver pathology had the following results:

	Scan shows	Scan shows	Total
	normal	abnormal	
True status	54	32	86
is normal			
True status	27	231	258
is abnormal			

a. What is the sensitivity of the liver scan for detecting abnormal liver pathology?

b. What is the specificity of the normal liver scan for detecting abnormal liver pathology?

c. Within the sample group of patients used for this study, what is the probability a patient actually has abnormal liver pathology if their liver scan shows abnormal?

d. To get a better estimate of the predictive value of the liver scan, what further information is important to know? Does this study appear to have a representative sample?

5. For the year of 1973, the University of California at Berkeley was involved in legal action relating to the claim that there was gender discrimination in graduate school admissions (see paper by Bickel et al. 1975 for details). The table below contains school-wide admission/denied numbers for 1973 for the pool of 12763 applicants.

	Admitted	Denied	Total
Male	3738	4704	8442
Female	1494	2827	4321

- a. What is the probability a randomly selected applicant was female?
- b. What is the probability a randomly selected applicant was admitted?
- c. What is the probability a randomly selected applicant was admitted if you knew the applicant was female?
- d. Do you think this is strong evidence of gender discrimination in the graduate school admission process?

Here is the breakdown by some of the graduate school departments (only the largest 6 departments are shown out of 101 total departments). Fill in the missing conditional probabilities of the form P(admit male given dept. A)= %M.admit and P(admit female given dept. A)= %F.admit.

Dept	M admit	M denied	M total	%M.admit	F admit	F denied	F total	%F.admit
А	512	313	825		89	19	108	
В	353	207	560	.63	17	8	25	.68
С	120	205	325	.37	202	391	593	.34
D	138	279	417		131	244	375	
Е	53	138	191	.28	94	299	393	.24
F	22	351	373	.06	24	317	341	.07

What do you think now about the claims of gender discrimination? What do you think was going on here?

Situations where probabilities seemingly "reverse" when a third variable (department) are taken into account are examples of a phenomenon known as Simpson's Paradox. In this case, it was determined that the "reversal" when department was taken into account was due to the fact that women had applied in larger numbers to departments that were harder to get into (fewer slots in proportion to number of applicants makes it harder even among qualified applicants) while men were applying to departments that were relatively easier to get into, resulting in a lower overall admission rate for women even if the department level bias appears to be in favor of women! 6. The following table contains data on number of eggs hatched and not for 3 different temperature settings for python eggs, as recorded by scientists studying environmental impact of temperature changes on python reproduction. As you answer the questions below, be sure to use clear notation for your events and probabilities.

	Hatched	Didn't hatch	Total
Cold temp	16	11	27
Neutral temp	38	18	56
Hot temp	75	29	104
Total	129	58	187

- a. What is the probability a randomly selected egg from this experiment hatched?
- b. Given each temperature setting (do a calculation for cold, neutral, and hot), what is the probability a randomly selected egg from that level hatched?

c. Verify Bayes' Rule for P(Neutral | Hatch) holds. In other words, apply Bayes' Rule to find the probability and verify it matches what you obtain using the table directly.