★ Counting Methods Summary

When you need to assign reasonable probabilities yourself, you need to be able to count the number of sample points in Ω. This can get complicated fast, but there are some useful rules to know and follow. This sheet contains useful summary information.

The mn-rule (or Fundamental Principle of Counting) states that with m elements, $a_1, a_2, ..., a_m$, and n elements, $b_1, ..., b_n$, you can form mn pairs containing one element from each set. Idea extends to any number of sets you want to combine in triples, quadruples, etc.

To count the number of ways to arrange r items from n depending on whether or not order of the r items is important and whether or not sampling is with or without replacement, the following chart is a helpful summary:

	Order is Important	Order is Not Important
With Replacement	n^r	choose$(n+r-1, r)$
Without Replacement	$P_r^n = \frac{n!}{(n-r)!}$	$C_r^n = \frac{n!}{(n-r)!r!} = choose(n, r)$

Let's start with selecting r objects from n WITH replacement when order matters. Selecting with replacement means that for each of the r selections, there are n options available. Because order is important, 432 should be counted as different from 234. Thus, there are n^r arrangements.

Permutations are used when order is important and sampling is done without replacement. The number of ordered arrangements of r objects out of n ($r \leq n$) is given by $P_r^n = n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$. Note that $0! = 1$ by definition.

Combinations are used when order is NOT important and sampling is done without replacement. The number of distinct subsets (or combinations) of size r that can be selected from n distinct objects $r \leq n$ is given by: $C_r^n = \frac{P_r^n}{r!} = \frac{n!}{(n-r)!r!}$=choose($n$,$r$).

Combinations can be extended to multinomial coefficients if you have more than 2 groups you are trying to split the objects into (r in and $n-r$ out is 2 groups for a combination). The number of ways of partitioning n distinct objects into k distinct groups consisting of $n_1, n_2, \ldots, n_k$ objects where each object appears in exactly one group, and $\sum_{i=1}^{k} n_i = n$ is given by: $N = \frac{n!}{n_1!n_2!n_3!\ldots n_k!}$.

The last situation covered in the table results in a combination coefficient, but doesn't have a neat reserved name like a combination or permutation does. This method is for when order is not important and sampling is WITH replacement (or, as an example, when you can't tell objects apart). The number of ways of making r selections from n objects when selection is made with replacement and order is not important is given by: choose($n+r-1$,r).

Thank you to Prof Wagaman for sharing this sheet.