Discrete distributions practice problems

1. A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that at least one accident will occur during any given month is 3/5. The number of accidents that occur in any given month is independent of the number of accidents that occur in all other months. Calculate the probability that there will be exactly three months with no accidents before a month in which at least one accident occurs.

2. Your textbook has some notes about R commands for the geometric and negative binomial distributions that you should pay close attention to. R works with those distributions based on counting the number of failures, NOT the trial number of the last success. For example: dgeom(12, 0.4) would compute the probability of having 12 failures before the first success when the probability of success is 0.4. The value of X from the geometric distribution we see in the text would be X=13, the first success is on the 13th trial. The command dnbinom(13, 4, 0.4) gives the probability of having 13 failures before the 4th success when probability of success in each trial is 0.4. This would correspond to X=17 for the negative binomial distribution (r=4, p=0.4) presented in your text. For the following situations, provide R code that you could use to compute the probability desired.

    a. $X \sim \text{Geo}(0.3)$. Compute $P(X = 6)$.

    b. $X \sim \text{Binomial}(10, 0.4)$. Compute $P(2 \le X \le 6)$.

    c. $X \sim \text{NegBinom}(4, 0.2)$. Compute $P(X = 20)$.

    d. $X \sim \text{Poisson}(8)$. Compute $P(X < 6)$.

3. Suppose your printer is acting up and you need to call the support line for assistance. The lines are busy 60 percent of the time, and you hang up if you don't get through immediately. Assume you keep calling until you get through to a person.

   a. What is the probability you successfully get through on the second try (but you did not make it through on the first try)?

   b. If you and a friend are both making calls (independently), what is the probability that a total of 4 tries will be needed before both of you are successful?

   c. What is the expected number of total tries for you and your friend to get through?

4. An accountant is checking audits on company accounts made by a junior associate. The associate makes minor mistakes on 20 percent of his audits and major mistakes in an additional 5 percent.

    a. If the accountant starts randomly checking the audits on company accounts, what is the probability she will have to go through exactly three good audits before finding the fourth one has a mistake (either major or minor)?

    b. What is the probability she will have to go through exactly 8 audits before finding one with a major mistake (on the 8th audit)?

    c. What are the mean (expectation), variance, and standard deviation of the number of audits she will have to go through before finding three with mistakes?

    d. If she focuses just on major mistakes and hasn't found any in the first 10 audits, what is the probability she sees at least 5 more without major mistakes before the first one with a major mistake?

5. Suppose you get phone calls in the evening, averaging about two per hour.
   a. What distribution might you use to model the number of phone calls you receive in an hour?

   b. If you take a 10-minute shower, what is the probability the phone rings at least once during that time?

   c. What is the longest shower you can take if you want the probability of no calls during that time to be at least 0.5?

6. A large club has to select a jury of 6 members to oversee an internal proceeding against another member. Out of all club members, 20 are considered eligible jurors due to eligibility constraints. Of the 20, 8 are women and 12 are men. Jury members are supposed to be chosen at random from all eligible jurors. Let the random variable X be the number of women that end up on the jury.
   a. What distribution does X have?

   b. If only one woman ends up on the jury (X=1), do you doubt the randomness of the jury selection? Find the probabilities of X being 0, 1, 2, 3, or 4.

   c. If jury selection is random, what is the expectation for the number of women selected for jury duty?