# Fall 2018 STAT 360 Exam 1 Review Problems

**The exam will be in class on Tuesday, October 16, and will cover Chapters 1-5.** You may use one sheet of notes (front and back of an 8.5x11in piece of paper), which is to be turned in with the exam. You may use a calculator during the exam; you may not use a phone or borrow a calculator from a classmate during the exam. Please turn off all phones and anything else that might beep or be a distraction.

<u>Practice exam problems</u>

1. Which is more likely: 9 heads in 10 tosses of a fair coin or 18 heads in 20 tosses?

2. On a large college campus, students get free access to copies of the school newspaper, the local newspaper, and a national newspaper. 82% read at least one paper, 42% read only the school newspaper, 18% read only the local newspaper, 6% read only the national newspaper, and 1% read all three papers. 52% read at least the school newspaper, and 5% read at least the school and national newspaper.
    a. Find the probability that a randomly selected student does not read any paper.
    b. Find the probability that a randomly selected student reads at least the local paper.
    c. Find the probability that a randomly selected student reads at least two papers.

3. According to the National Center for Statistics and Analysis, in 2004, 28% of the drivers involved in fatal crashes were between 20 and 29 years of age. Further, 39% of the 20- to 29-year-old drivers involved in fatal crashes had a blood alcohol level of at least 0.01. In what percentage of fatal crashes were the drivers both between 20 and 29 years of age and found to have a blood alcohol level above 0.01?

4. If A and B are independent events, prove that A and $B^c$ are also independent.

5. Let X be a binomial random variable with n = 10 trials and p = 0.2 probability of success. Calculate P (X > 2).

6. A study to see if a new test can detect a disease had the following results:

| | Test positive for disease | Test negative for disease | Total |
|---|---|---|---|
| Disease is present | 91 | 9 | 100 |
| Disease is not present | 27 | 873 | 900 |

   a. What is the sensitivity of the new test for detecting the disease?
   b. What is the specificity of the new test?
   c. Within the sample group used for this study, what is the probability a patient actually has the disease if the test is positive?

7. Prostate-specific antigen (PSA) is a commonly used marker for detection of prostate cancer. Its sensitivity is 0.80, and its specificity is 0.59. Assume that 0.1% of men in the US getting tested have prostate cancer.
   a. What is the probability that someone with prostate cancer will have a negative result when using PSA?
   b. What is the probability that someone without prostate cancer will have a positive result when using PSA?
   c. What is the predictive value of this marker (probability of having disease given a positive test)?

8. Let X be a binomially distributed random variable with n trials and probability p of success on each trial. For which value of k is $P(X = k)$ maximized? (Hint: Consider the inequality $P(X=k) < P(X=k+1)$ to determine when the probability stops increasing and begins decreasing with k.)

9. When taping a television commercial, the probability is 0.40 that a certain actor will get his lines straight on any one take. What is the probability that he will get his lines straight for the first time on the third take?

10. The international student organization is putting together a committee from the thirteen members of the executive board (five Chinese, four Kenyans, two Australians, and two Canadians).
    a. Find the probability that all four countries are represented if a committee of size four is chosen at random.
    b. Answer the same question if a subcommittee of size five is chosen.

11. Three identical fair coins are thrown simultaneously until all three show the same face. What is the probability that they are thrown more than three times?

12. A car salesman must make 3 sales each day. Experience suggests that if he chats with a customer, the probability that the customer will purchase a car is 0.2.
    a. What is the probability that the salesman will have to chat with at least 5 customers to make 3 sales?
    b. What is the expected value of the number of customers he must chat with to make 3 sales?

13. Suppose that in a certain city the number of muggings can be approximated by a Poisson process with $\lambda = 4$ per month.
    a. Calculate the probability of 3 muggings in one month.
    b. Write a mathematical expression for the probability of 45 muggings in a year.

14. An auditor checking the accounting practices of a small firm randomly samples 4 accounts from a list of 12. Find the probability that the auditor sees at least one past-due account under each of the following conditions:
    a. There are 2 past-due accounts in the list of 12.
    b. There are 6 past-due accounts in the list of 12.
    c. There are 8 past-due accounts in the list of 12.

15. You receive texts from two friends. Suppose that the total number of texts per day from them is Poisson with parameter $\lambda$. Each text has probability $p_1$ of being from your first friend, and probability $p_2$ of being from your second friend, where $p_1 + p_2 = 1$. Find the average number of texts per day from your first friend.

16. Let X and Y be jointly distributed discrete RVs with joint distribution given in the following table:

| Y \ X | 1 | 2 |
|---|---|---|
| 0 | 0.2 | 0.05 |
| 1 | 0.2 | 0.1 |
| 2 | 0.1 | 0.2 |
| 3 | 0.05 | 0.1 |

a. Find the marginal distribution of X.
b. Determine $E(X^2 + 3X)$.
c. Find the conditional distribution of Y given that X = 2.
d. Find E(Y | X = 2).

17. A 10-acre area has 40 raccoons. Ten of these raccoons were captured, tagged, and released. Another sample of 20 raccoons was taken 5 days later. Let X denote the number of tagged raccoons in this new sample.
   a. What distribution does X have?
   b. Write a mathematical expression giving the probability that no more than 5 of those captured in the new sample have a tag.

18. Consider a deck of 20 cards with 10 purple cards numbered 1 to 10 and 10 white cards, also numbered 1 to 10. A single card will be drawn at random from the deck. Define the following events:

- A – an even numbered card is drawn from the deck
- B – a white card is drawn from the deck
- C – a card with a number (strictly) less than 5 is drawn from the deck

   a. Find the probability of each event, A, B, and C.
   b. Are A and B independent events? Justify your answer.
   c. Find the probability that the card drawn is not white or the card does not have a number less than 5 on it (or both not white and not less than 5).
   d. Find the probability that the number on the card is even and the card is white.

19. Let X and Y be i.i.d. RVs with finite expectation and variance. Evaluate Cov(X +Y, Y).

20. Consider a garden with 12 strawberry plants and 18 blueberry plants.

   a. Randomly select 12 plants from the garden to pick berries from. What is the probability you pick from exactly 7 of the strawberry plants? (Set up, but do not evaluate).
   b. You randomly select 10 of the 30 plants to treat with neem oil. Let $X$ denote the number of blueberry plants in this sample of 10 plants to treat. What distribution does X have? Be sure to fully specify all necessary parameters.
   c. How many blueberry plants on average would be treated?
   d. Set up (but do not evaluate) an expression for the probability that between five and nine blueberry plants (inclusive) are treated with neem oil.

Brief Solutions for Exam 1 Review Problems

Note: these are provided so you can check that you are on the right track. On the exam, be sure to show all work and clearly explain your solution method. If you have the right approach written out but make an arithmetic mistake, I will then still be able to give you most of the credit.

Please report any errors in these answers to tleise@amherst.edu. Thank you!

1. Probabilities are .00976 and .00018, respectively.
2. (a) 0.18 (b) 0.3 (c) 0.16
3. 0.1092
4. $P(B|A)=P(B)$, so $P(B^c|A)=1-P(B|A)=1-P(B)=P(B^c)$
5. 0.322
6. (a) 0.91 (b) 0.97 (c) 0.771
7. (c) 0.002
8. Round down $(n+1)p$
9. 0.144
10. (a) 0.112 (b) 0.280
11. 27/64
12. (a) 0.973 (b) 15
13. (a) 0.195 (b) $e^{-48}48^{45}/45!$
14. (a) 0.576 (b) 0.970 (c) 0.998
15. $\lambda p_1$
16. (a) $P(x=1)=0.55$, $P(X=2)=0.45$
    (b) 6.7
    (c) $P(Y=0|X=2)=1/9$, $P(Y=1|X=2)=2/9$, $P(Y=2|X=2)=4/9$, $P(Y=3|X=2)=2/9$
17. (a) X~HyperGeo(10,30,20)
    (b) $\sum_{k=0}^{5} P(X=k) = \sum_{k=0}^{5} \binom{10}{k}\binom{30}{20-k}/\binom{40}{20}$
18. (a) $P(A)=1/2$, $P(B)=1/2$, $P(C)=2/5$
    (b) Yes, A and B are independent because $P(AB)=1/4=P(A)P(B)$
    (c) 0.8
    (d) 0.25
19. Expand out using definition to derive $Cov(X+Y,Y)=Cov(X,Y)-Var(Y)$
20. (a) $\binom{12}{7}\binom{18}{5}/\binom{30}{12}$
    (b) X~HyperGeo(18,12,10)
    (c) 6
    (d) $\sum_{k=5}^{9}\binom{18}{k}\binom{12}{10-k}/\binom{30}{10}$