

Exponential and joint distributions practice problems

1. Suppose that the number of phone calls you receive during the evening is described by a Poisson distribution with an average of two calls per hour. Suppose you know that exactly one call has occurred between 8pm and 9pm.
 - a. What is the probability the phone call occurred between 8 and 8:15pm?
 - b. What is the probability the phone call occurred after 8:30pm if you know it happened before 8:45pm?
 - c. Suppose it's now 9pm. What is the probability that you receive at least one call in the next half hour?

2. Suppose X and Y have joint density given by $f(x,y) = 2x$ for $0 \leq x \leq 1, 0 \leq y \leq 1$, and 0 otherwise.
 - a. Find $E[X]$, $E[Y]$, and $E[XY]$.

 - b. Find the covariance of X and Y .

 - c. Find the marginal distribution of X and the marginal distribution of Y .

 - d. Are X and Y independent? Justify for your answer.

3. Let X and Y be jointly continuous RVs with pdf given by $f(x, y) = k(x - y)$ for $0 \leq y \leq x \leq 1$, and 0 otherwise.
- Find the value of k that makes this a valid pdf.
 - Find the marginal pdf of X .
 - Find the cumulative probability $F(5/6, 1/2)$.
 - Find $P(X \geq 3/4, Y \geq 1/4)$.
 - Find $P(X \geq 2Y)$.
 - Find $P(X + Y \leq 1)$.